Robust and Optimal Control Approach for Continuous Stirred Tank Reactors

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***Abstract*— The aim of this paper is design of a robust static output feedback and optimal controllers for a Continuous-time Stirred Tank Reactor (CSTR). some of CSTR's parameters are uncertain and designed controller is able to stabilize CSTR with four uncertain parameters.in this paper, the nonlinear model of CSTR is considered and then controller designed with linearization and solving two Linear Matrix Inequalities (LMI). the approach is tested with representative example. Comparison between this paper results and previous results shown, settling time is improved by better solving strategy.**

***Index Terms*— Robust control, CSTR, LMI approach, yalmip.**

I. Introduction

CHEMICAL and biochemical reactors are key units in chemical, pharmaceutical and food industries, and especially exothermic jacketed continuous stirred tank reactors(CSTRs) are used for production of various important materials [1]. Chemical reactors processing is connected with many different uncertainties. These uncertainties can cause poor performance or instability of closed-loop control system. Application of robust control approach can be one of the possibilities how to overcome all these problems [2]. In this paper, conditions for robust stabilization of nonlinear continuous-time Invariant systems via static output feedback are presented. The focus of the present work is the application of this new technique to the calculation of robust stability and robust performance for a nonlinear CSTR model. Parameters are taken from the [3], but in this paper unlike [3], static output feedback is derived from two outputs and simulation is gone yalmip and Matlab/Simulink.

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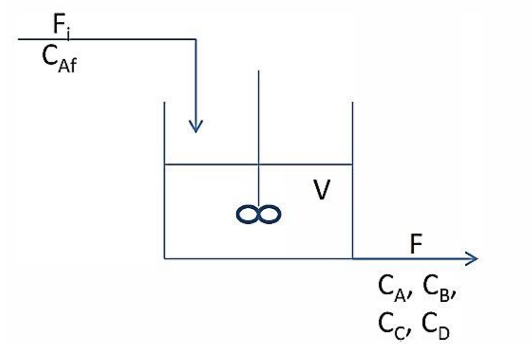

Fig. 1. Consider a CSTR.

II. CSTR Model Description

Consider a CSTR, shown in Figure 1, that is operated at constant temperature, i.e., isothermal. A stream of pure "A" is fed into the reactor at a volumetric flow of (Fi), where it reacts to form components "B", "C" according to the following two irreversible reactions:

$$A \xrightarrow{k1} B \xrightarrow{k2} C$$

$$k_j = k_{j0} e^{(\frac{-E_j}{RTr})} \quad (1)$$

K_0 are pre-exponential factors, E are activation energies, R is the gas constant.

III. Problem Formulation

The dynamic mathematical model of the CSTR is described by the mass and energy balances given by (2)– (5).

$$\frac{dc_A}{dt} = -\left(\frac{q_r}{V_r} + k_1 + k_2\right) c_A + \frac{q_r}{V_r} c_{Af} \quad (2)$$

$$\frac{dc_B}{dt} = -\frac{q_r}{V_r} c_B + k_1 c_A + \frac{q_r}{V_r} c_{Bf} \quad (3)$$

$$\frac{dT_r}{dt} = -\frac{h_1 k_1 + h_2 k_2}{\rho_r c_{pr}} c_A + \frac{q_r}{V_r}\left(T_{rf} - T_r\right) + \frac{A_h U}{V_r \rho_r c_{pr}} (T_c - T_r) \quad (4)$$

$$\frac{dT_c}{dt} = \frac{q_c}{V_c}\left(T_{cf} - T_c\right) + \frac{A_h U}{V_c \rho_c c_{pc}} (T_r - T_c) \quad (5)$$

TABLE I
THE VALUES OF ALL PARAMETERS AND FEED VALUES

Variable	Unit	Value
V_r	m	0.23
V_c	m^3	0.21
ρ_r	Kgm^{-3}	1020
ρ_c	Kgm^{-3}	998
c_{Pr}	$KjKg^{-1}K^{-1}$	4.020
c_{Pc}	$KjKg^{-1}K^{-1}$	4.182
A	m^2	1.51
U	$KJm^{-2}min^{-1}K^{-1}$	42.8
g1=E1/R	K	9850
g2=E2/R	K	22019
C_{Af}	Kmol m^{-3}	4.22
C_{Bf}	Kmol m^{-3}	0
T_{rf}	K	310
T_{cf}	K	288
q_r^s	m^3mm^{-1}	0.015
q_c^s	m^3mm^{-1}	0.004

TABLE II
UNCERTAIN PARAMETERS

Parameter	Unit	Minimum value	Maximum value
h1	$KJKmol^{-1}$	$-8.8*10^4$	$-8.4*10^4$
h2	$KJKmol^{-1}$	$-5.7*10^4$	$-5.3*10^4$
K10	mm^{-1}	$1.5*10^{11}$	$1.6*10^{11}$
K20	mm^{-1}	$495*10^{26}$	$12.15*10^{26}$

Here, t is time, c are concentrations, T are temperatures, V are volumes, ρ are densities, *Cp* are specific heat capacities, q are volumetric flow rates, h are reaction enthalpies, Ah is the heat transfer area and U is the heat transfer coefficient. The subscripts denote. r the reactant mixture, .c the coolant, f feed values and the superscript. s the steady-state values. The values of all parameters and feed values are in table 1. Suppose further the CSTR has 4 uncertain parameters. They are shown in the table 2. [1]
The main operating point for the nominal model is

$[C_A^S, C_B^S, T_r^S, T_c^S]$ =[1.8614,1.0113,338.41,328.06]

IV. THEORETICAL ANALYSIS

Consider an uncertain linear Time Invariant (LTI) system in the form

$$\frac{dx(t)}{dt} = A(\sigma)x(t) + B(\sigma)u(t)$$
$$\mathrm{y(t)} = \mathrm{c}(\sigma)\,\mathrm{x(t)} \tag{6}$$

δ is vector of uncertain parameters. The static output feedback $u(\mathrm{t}) = F\ \mathrm{y(t)}$ can be formulated as follows.

$$\frac{dx(t)}{dt} = (A(\sigma) + B(\sigma)FC(\sigma))\,\mathrm{x(t)} \quad \mathrm{x(t_0)} = x_0 \tag{7}$$

Considering Lyapunov theorem and minimization of the following cost function then the following statements are equivalent [4].

$$J = \frac{1}{2}\int_0^\infty (x^T(t)Qx(t) + u^T(\mathrm{t})\,\mathrm{Ru(t)})\,\mathrm{dt} \qquad \mathrm{i=1, \ldots, n} \tag{8}$$

$$J = \frac{1}{2}\int_0^\infty (x^T(t)Qx(t) + u^T(\mathrm{t})\,\mathrm{Ru(t)})\,\mathrm{dt} \tag{9}$$

$$(A_i + B_iFC_i)^T P + P\,(A_i + B_iFC_i) + Q + C_i^T F^T RFC_i < 0 \tag{10}$$

By solving two Linear Matrix Inequalitis (LMI), F is obtained.

$$\begin{bmatrix} SA_i^T + A_iS - B_iR^{-1}B_i^T & S\sqrt{Q} \\ \sqrt{Q}S & -I \end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix} -R & B_iP + RFC_i \\ (B_iP + RFC_i)^T & -\varnothing_i \end{bmatrix} < 0 \tag{12}$$

Where

$$S = P^{-1}$$
$$\phi_i = -\left(A_i^T P + P\,A_i - PBR^{-1}B_i^T P + Q\right)$$

V. LINEARIZATION AND ROBUST CONTROL

Equations (2-5) are linearized and, system state space is obtained. Design of a robust stabilizing controller is based on having a linear state space model. Linear Equations are as follows.

$$\mathrm{A} = \begin{pmatrix} \frac{\partial \dot{c_A}}{\partial c_A} & \frac{\partial \dot{c_A}}{\partial c_B} & \frac{\partial \dot{c_A}}{\partial T_r} & \frac{\partial \dot{c_A}}{\partial T_c} \\ \frac{\partial \dot{c_B}}{\partial c_A} & \frac{\partial \dot{c_B}}{\partial c_B} & \frac{\partial \dot{c_B}}{\partial T_r} & \frac{\partial \dot{c_B}}{\partial T_c} \\ \frac{\partial \dot{T_r}}{\partial c_A} & \frac{\partial \dot{T_r}}{\partial c_B} & \frac{\partial \dot{T_r}}{\partial T_r} & \frac{\partial \dot{T_r}}{\partial T_c} \\ \frac{\partial \dot{T_c}}{\partial c_A} & \frac{\partial \dot{T_c}}{\partial c_B} & \frac{\partial \dot{T_c}}{\partial T_r} & \frac{\partial \dot{T_c}}{\partial T_c} \end{pmatrix} \quad . \quad B = \begin{pmatrix} \frac{\partial \dot{c_A}}{\partial q_r} & \frac{\partial \dot{c_A}}{\partial q_c} \\ \frac{\partial \dot{c_B}}{\partial q_r} & \frac{\partial \dot{c_B}}{\partial q_c} \\ \frac{\partial \dot{T_r}}{\partial q_r} & \frac{\partial \dot{T_r}}{\partial q_c} \\ \frac{\partial \dot{T_c}}{\partial q_r} & \frac{\partial \dot{T_c}}{\partial q_c} \end{pmatrix} \tag{13}$$

$$\frac{\partial \dot{c_A}}{\partial c_A} = -\left(\frac{q_r}{V_r} + k_1 + k_2\right)$$
$$\frac{\partial \dot{c_A}}{\partial c_B} = 0$$
$$\frac{\partial \dot{c_A}}{\partial T_r} = -\frac{c_A}{(T_r)^2}(g_1k_1 + g_2k_2)$$
$$\frac{\partial \dot{c_A}}{\partial q_r} = -\frac{c_A}{V_r} + \frac{c_{Af}}{V_r}$$
$$\frac{\partial \dot{c_A}}{\partial q_c} = 0$$
$$\frac{\partial \dot{c_B}}{\partial c_A} = k_1$$
$$\frac{\partial \dot{c_B}}{\partial c_B} = -\frac{q_r}{V_r}$$

$$\frac{\partial \dot{c_B}}{\partial T_r} = \frac{c_A g_1}{(T_r)^2} k_1$$
$$\frac{\partial \dot{c_B}}{\partial T_c} = 0$$
$$\frac{\partial \dot{c_B}}{\partial q_r} = -\frac{c_B}{V_r} + \frac{c_{Bf}}{V_r}$$
$$\frac{\partial \dot{c_B}}{\partial q_c} = 0$$
$$\frac{\partial \dot{T_r}}{\partial c_A} = -\frac{h_1 k_1 + h_2 k_2}{\rho_r c_{pr}}$$
$$\frac{\partial \dot{T_r}}{\partial c_B} = 0$$
$$\frac{\partial \dot{T_r}}{\partial T_r} = -c_A \frac{h_1 g_1 k_1 + h_2 g_2 k_2}{\rho_r c_{pr} (T_r)^2} - \frac{q_r}{V_r} - \frac{A_h U}{V_r \rho_r c_{pr}}$$
$$\frac{\partial \dot{T_r}}{\partial T_c} = \frac{A_h U}{V_r \rho_r c_{pr}}$$
$$\frac{\partial \dot{T_r}}{\partial q_r} = \frac{T_{rf} - T_r}{V_r}$$
$$\frac{\partial \dot{T_r}}{\partial q_c} = 0$$
$$\frac{\partial \dot{T_c}}{\partial c_A} = 0$$
$$\frac{\partial \dot{T_c}}{\partial c_B} = 0$$
$$\frac{\partial \dot{T_c}}{\partial T_r} = \frac{A_h U}{V_c \rho_c c_{pc}}$$
$$\frac{\partial \dot{T_c}}{\partial T_c} = -\frac{q_c}{V_c} - \frac{A_h U}{V_c \rho_c c_{pc}}$$
$$\frac{\partial \dot{T_c}}{\partial q_r} = 0$$
$$\frac{\partial \dot{T_c}}{\partial q_c} = \frac{T_{cf} - T_c}{V_c}$$

The matrices of the nominal linearized model in the main operating are

$$A_0 = \begin{bmatrix} -0.1479 & 0 & -0.0226 & 0 \\ 0.0354 & -0.0652 & 0.0057 & 0 \\ 1.3766 & 0 & 0.2119 & 0.07 \\ 0 & 0 & 0.0737 & -0.0928 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 10.2546 & 0 \\ -4.3968 & 0 \\ -123.5131 & 0 \\ 0 & -190.7612 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The parameters of matrices Q, R have been chosen according to the values of state variables and control inputs.

R=diag (10,10)
Q=diag $(10^{-3},10^{-3},10^{-4},10^{-4})$

The eigenvalues of A_0 are -0.0652, 0.1191, -0.0740 + 0.0310i, -0.0740 - 0.0310i and shown this model is unstable. we have obtained 16 linearized models for 4 uncertain parameters. It is important to find a robust static output feedback, which would be able to stabilize the whole uncertain system with the guaranteed cost expressed by (9), For solving the LMIs, the YALMIP MATLAB was used. The parameters of the controllers are as follows.

$$F = \begin{bmatrix} 0.0208 & 0.0052 \\ 0.0052 & 0.0025 \end{bmatrix}$$

The eigenvalues of close loop are on the left of jw axis, in 16 cases, shown this model is stable with this static output feedback. Simulation results obtained with the robust static feedback controller designed is shown in Fig. 2 and Fig.3

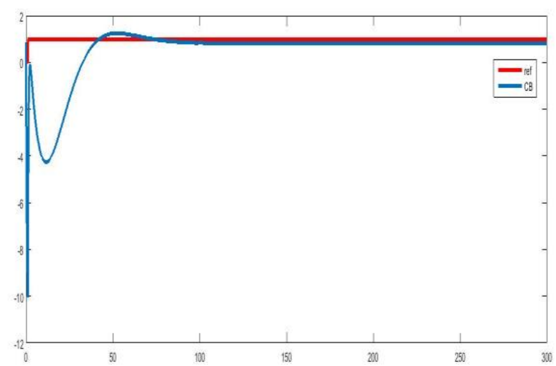

Fig. 2. Simulation results obtained with the robust static feedback controller designed.

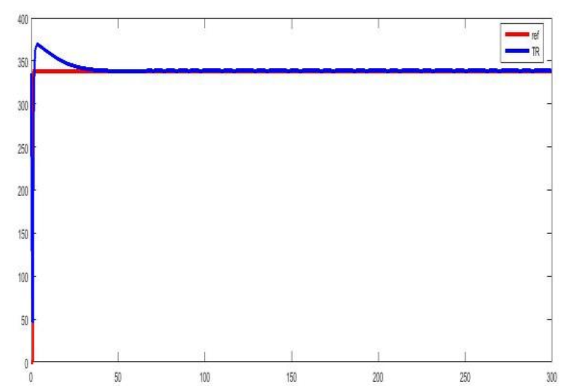

Fig. 3. Simulation results obtained with the robust static feedback controller designed.

VI. Conclusion

In this paper, some of CSTR's parameters are uncertain and designed robust controller is able to stabilize CSTR and guaranteed cost function. The robust controllers design is converted to solving of LMI problems. The designed robust controllers are able to stabilize the exothermic CSTR for the entire operating area not only for a single operating point. Comparison between this paper results and results from [1], [3] shown , settling time is improved by Better solving strategy.

VII. REFERENCES

[1] M. Bakošováa, A. Mészárosa, J. J. Klemešb, and J. Oraveca . Robust and Optimal Control Approach for Exothermic Reactor Stabilization. Theoretical Foundations of Chemical Engineering, , Vol. 46, No. 6, pp. 740–746.2012

[2] Alvarez-Ramirez, J. and R. Fermat,, Robust PI stabilization of a class of chemical reactors Systems Control Lett. 38, 219-225.1999

[3] Monika Bakošováa,, Dalibor Puna and Alojz Meszfiros. Robust Controller Design for a Chemical Reactor. European Symposium on Computer Aided Process Engineerin. Elsevier,2005
,
[4] Vesel., Robust output feedback controller design for linear parametric uncertain systems, Journal of Electrical Engineering, 53, 117-125.2002

[5] Vasičkaninová, M. Bakošová, Ľ. Čirka, M. Kalúz (2015). Comparison of Robust Control Techniques for Use in Continuous Stirred Tank Reactor Control. IFAC-PapersOnLine 48-14 - 284–289.2015

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